Math 51 hand-in problems, fall 2017

## Sections 0.1-0.3, due Friday Sept 1

(1) If $f(x) = x^2$ and $h \neq 0$, compute $\dfrac{f(2+h) - f(2)}{h}$ in simplified form.

(2) Describe the domains of the functions $f(x) = \dfrac{\sqrt{x}}{x-1}$ and $g(x) = \dfrac{\sqrt{x+2}}{x\sqrt{x+1}}$.

(3) Sketch the graph of $f(x)$, where $f(x) = \begin{cases} x+1 & x \geq 0 \\ 0 & x < 0. \end{cases}$

(4) Sketch the graph of a function $f(x)$ satisfying (a) $f(x) \geq 0$ for $x \geq 1$ and for $x \leq -2$, (b) $f(x) < 0$ for $x$ between $-2$ and 1, and (c) $f(0) = -1$.

(5) Sketch the graph of $f(x) = |x+1|$.

(6) Let $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{2}{x-1}$. Express $f(x) + g(x)$ and $f(x) - g(x)$ as rational functions, simplified.

(7) Let $f(x) = x^2$ and $g(x) = \frac{x}{x+1}$. Write in simplified form $f(g(x))$, $g(f(x))$, and $g(g(x))$. Label your answers.

## Sections 0.4-0.6, due Wednesday Sept 6.

(1) Use the quadratic formula to solve the equation $2x^2+5x+1 = 0$.

(2) Find the points of intersection of the curves $y = 2x^2 + 4x + 1$ and $y = x + 3$.

(3) Compute $.0001^{1/4}$, $8^{-2/3}$, and $(-3)^{-2}$.

(4) Simplify each of the following.

$$\frac{-x^2y^3}{-xy}, \quad \frac{(-8x^5)^{2/3}}{\sqrt[3]{x}}, \quad \sqrt[4]{x} \cdot \sqrt{x}.$$

(5) A rectangular box with no top has sides costing \$1 per square foot and base costing \$2 per square foot. Assign variables to the three dimensions, and write a formula for the cost in terms of those dimensions.

(6) The cost of manufacturing $x$ items is $100 + 3x$ dollars. You sell them for \$5 apiece. Write a formula for the profit if you make and sell $x$ items. How many must you make in order to break even?

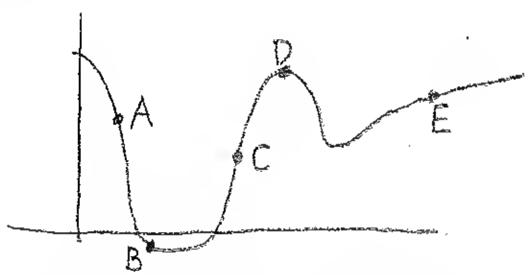
**Sections 1.1-1.2, due Monday Sept 11**

(1) Write the equation of straight lines such that
  a. $(1, 3)$ and $(4, -1)$ lie on the line,
  b. $x$-intercept is 4, and $y$-intercept is $-2$.

(2) Line $\ell_1$ has slope 2 and $y$-intercept 3. Line $\ell_2$ is perpendicular to line $\ell_1$ and passes through the point $(-1, 2)$. Write the equations of each and plot them on the same set of axes.

(3) Sue can sell 1000 gallons of gasoline at \$2.20 per gallon, and 850 gallons at \$2.35 per gallon. Assume sales are linearly related to price. Let $G(x)$ denote the number of gallons sold at a price of \$x per gallon. Write the formula for $G(x)$. What price will allow Sue to sell 1500 gallons?

(4) For each point on the indicated graph, tell whether the slope is very positive, slightly positive, zero, slightly negative, or very negative.




(5) Write the equation of the tangent line to the curve $y = x^2$ at the point where $x = 3$ and at the point where $x = -3$.

(6) The slope of the curve $y = x^3$ at the point $(x, y)$ is $3x^2$. Find the points on the curve where the tangent line is parallel to the line $y = x - 1$.

**Sections 1.3-1.4, due Friday Sept 15**

(1) For $f(x) = \frac{1}{\sqrt[3]{x}}$, find $f(8)$, $f'(8)$, and the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 8$.

(2) Sketch the curve $y = \sqrt{x}$. Find the equation of the tangent line to the curve which has slope $1/8$, and include this tangent line in your sketch. Show the $y$-intercept of the tangent line in your graph.

(3) Apply the "three-step method" (p. 78) to compute the derivative of $f(x) = -2x^2$.

(4) Determine which of the following limits exist, and compute those that do.

$$\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6}, \qquad \lim_{x\to -2} \frac{x^2-4}{x^2-5x+6}, \qquad \lim_{x\to 3} \frac{x^2-4}{x^2-5x+6}.$$

(5) Use limits to compute the derivative of $f(x) = \sqrt{x-1}$.

(6) Compute the following limits. Show your work.

$$\lim_{x\to\infty} \frac{3x-2}{x^2+1}, \qquad \lim_{x\to\infty} \frac{3x^2-2}{x^2+1}.$$

**Sections 1.5-1.6, due Wednesday Sept 20**

(1) Determine whether each of the following functions $f$ is continuous and/or differentiable at $x = 2$.

$$f(x) = \frac{1}{x}, \qquad f(x) = |x-2|, \qquad f(x) = \begin{cases} x & x \neq 2 \\ 1 & x = 2. \end{cases}$$

(2) If possible, for each of the following, define $f(x)$ at the exceptional point $x$ in a way that makes $f(x)$ continuous at $x$.

$$f(x) = \frac{x^2 - 5x + 6}{x - 2}, \; x \neq 2, \qquad f(x) = \frac{\sqrt{x+4} - 2}{x}, \; x \neq 0.$$

(3) Find values of $a$ and $b$ so that $f(x) = \begin{cases} x & x \leq 2 \\ ax^2 + b & x > 2 \end{cases}$ is differentiable for all $x$.

(4) Differentiate each of the following:

$$y = \frac{1}{x^2 + x}, \qquad y = \sqrt{x^3 + 1}, \qquad y = \left(x + \tfrac{1}{x}\right)^3.$$

(5) Find the equation of the tangent line of the curve $y = (x^2 + 1)^3$ at the point where $x = 1$.

(6) If $g(x) = 3\sqrt{f(x)}$, find $g(1)$ and $g'(1)$ if $f(1) = 9$ and $f'(1) = -2$.

## Sections 1.7-1.8, due Monday Sept 25

(1) Find the first and second derivatives of (a) $y = \sqrt{3x - 2}$; (b) $P = (2t - 1)^3 + t^2$.

(2) Find $\frac{d}{dt}\left(\frac{dy}{dt}\right)\Big|_{t=3}$ if $y(t) = t^2 - \frac{1}{t-1}$.

(3) The profit from producing and selling $x$ units of a product is given by $P(x) = 5x - 10 - .01x^2$.

a. What is the additional profit if production is increased from 30 to 31?

b. What is the marginal profit when $x = 30$?

(4) If $f(t) = t^2 + 4t - 1$, find (a) the average rate of change of $f(t)$ over the interval from 4 to 4.5, and (b) the instantaneous rate of change of $f$ at $t = 4$.

(5) A toy rocket is fired straight up and has height $s(t) = 240t - 16t^2$ feet after $t$ seconds.

a. What is its velocity after $t$ seconds?

b. What is its acceleration after $t$ seconds?
c. When does the rocket reach its maximum height?
d. What is its maximum height?
e. When does it hit the ground?
f. What is its velocity just before it hits the ground?

(6) If $f(20) = 8$ and $f'(20) = 2$, what are the approximate values of $f(21)$, $f(20.5)$, $f(19.8)$, and $f(18)$?

**Sections 2.1-2.2, due Friday Sept 29**

(1) Describe the graph in Figure 1 in terms of each of the six categories in the box on page 137.

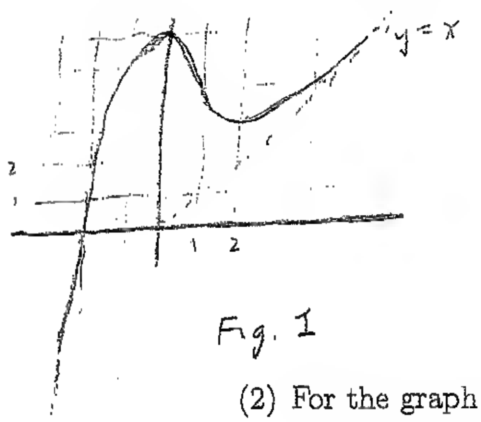



Fig. 1

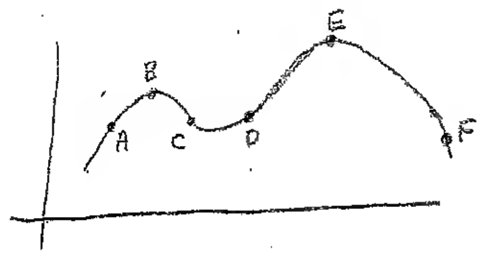



Fig. 2

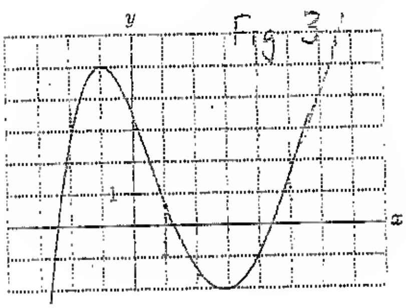
(2) For the graph in Figure 2, answer the following:
a. At which of the labeled points is the function increasing?
b. At which of the labeled points is the function decreasing?
c. At which of the labeled points is the function concave up?
d. At which of the labeled points is the function concave down?
e. Which labeled point has the largest slope?
f. Which labeled point has the most negative slope?

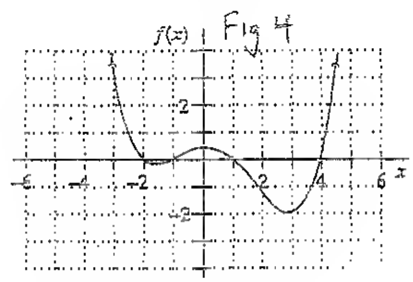
(3) At 8AM the temperature is 60° and rising at an increasing rate. At noon, it is still rising but at a decreasing rate. The maximum temperature of 80° occurs at 3PM. The temperature decreases from 4PM until midnight. The rate of decrease is largest (i.e.,

most negative) at 9PM. Sketch a graph of the temperature as a function of time.

(4) Sketch a graph $y = f(x)$ such that $f(x)$ is defined for $0 \le x \le 10$, $f'(x)$ is negative for $x$ between 1 and 4, and positive for $x < 1$ and $x > 4$, $(1,5)$ and $(4,1)$ lie on the graph, and $f''(x) < 0$ for $x < 2$ and is positive for $x > 2$.

(5) Figure 3 gives the graph of $f'(x)$. Assume the behavior at each end continues indefinitely. Tell the intervals on which $f$ is (a) increasing, (b) decreasing, (c) concave up, (d) concave down.




(6) Figure 4 gives the graph of $f(x)$. Sketch a graph of $f'(x)$.

## Sections 2.3-2.4, due Monday Oct 9

(1) The function $f(x) = -\frac{1}{3}x^3 - 2x^2 + 5x$ has one relative maximum and one relative minimum. Plot these points and check the concavity at each. Using only this information, sketch the graph.

(2) Sketch the graph of $f(x) = 2x^3 + 3x^2 + 1$, indicating all relative extreme points and inflection points. Show your work.

(3) Sketch the graph of $f(x) = x^4 - 4x^3$, indicating all relative extreme points and inflection points. Show your work.

(4) Sketch the graph of $f(x) = x + \frac{4}{x}$ for $x > 0$, indicating relative extreme points, inflection points, and asymptotes. Show your work.

(5) Sketch the graph of $f(x) = 2\sqrt{x} - x$, indicating relative extreme points and inflection points. Show your work.

(6) Find $a$, $b$, and $c$ so that the graph of $f(x) = ax^2 + bx + c$ passes through the point $(1, 1)$ and has a relative maximum or minimum at $(2, -1)$. Which is it–max or min?

## Sections 2.5-2.6, due Friday Oct 13

(1) Find the positive values of $x$ and $y$ that minimize $x + 4y$ subject to $xy = 36$.

(2) A rectangular garden with area 96 square feet is to be surrounded on one side by a brick wall costing \$10 per foot and on three sides by a fence costing \$5 per foot. What are the dimensions of the garden that minimize the cost?

(3) Points A and B on an east-west highway are 12 miles apart. City $A'$ is 6 miles north of point A, and city $B'$ is 4 miles north of point B. Point C on the highway is $x$ miles from point A in the direction of B. What value of $x$ minimizes the total of the diagonal distances from point C to the two cities? (See Figure 18 on page 170, with 11 changed to 12.)

(4) You can sell 5000 items in a year. You wish to get them in equal orders of $x$ items and sell them at a uniform rate. The carrying cost is \$4 per item, and the cost of placing an order is \$10. How many orders should you place, and what should be their size, in order to minimize the cost? Your answers will not be integers. In real life, you would round off.

(5) A rectangular corral of 100 square meters is to be fenced off and then divided by a fence into two sections, as shown in Figure 7b on page 176. Find the dimensions of the corral so that the amount of fencing required is minimized.

(6) An open rectangular box is to be constructed by cutting square corners out of a 16- by 12-inch piece of cardboard and folding

up the flaps. See Figure 9 on page 176. Find the value of $x$ for which the volume will be as large as possible.

**Sections 3.1-3.2, due Friday Oct 20**

(1) Differentiate the following functions. Simplify your answer to the second one.

$$y = (x^2+2)(x^3+2x+1)^8, \qquad y = \frac{x}{(x^2+1)^3}$$

(2) Find all $x$ such that $\frac{dy}{dx} = 0$ if $y = \dfrac{(x^2-2)^3}{(x-2)^2}$.

(3) Find formulas for the second and third derivatives of $y = f(x)g(x)$ in terms of the values and various derivatives of $f$ and $g$.

(4) Let $f(x) = \frac{1}{2+\sqrt{x}}$ and $g(x) = \frac{2}{x}$. Compute $\frac{d}{dx}(f(g(x))$ in two ways. First by using the formula in the box on page 203, and second by writing a simplified formula for $f(g(x))$ and then differentiating that.

(5) Find $\left.\frac{dy}{dt}\right|_{t_0=0}$ if $y = \sqrt{x+2}$ and $x = \sqrt{t+4}$.

(6) Work a version of problem 50 on page 207 for a snake which is 0.5 meters long and is growing at a rate of 0.15 meters per year.

**Sections 3.3-4.1, due Wednesday Oct 25**

(1) Find $\frac{dy}{dx}$ by implicit differentiation if $x^2 + xy - y^2 = 4$.

(2) Find the slope of the lemniscate $2(x^2+y^2)^2 = 25(x^2-y^2)$ at the point $(3,1)$. The graph resembles that in Figure 6 on page 216. Sketch the graph and find the $x$-value of the point at the extreme right on the graph, and then indicate roughly where the point $(3,1)$ lies on the curve.

(3) A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

(4) Two cars start moving from the same point. One travels south at 60 mph and the other travels east at 25 mph. At what rate is the distance between the two cars increasing two hours later?

(5) A street light is mounted at the top of a 15 foot pole. A man 6 feet tall walks away from the pole at a speed of 5 feet per second along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole? (Hint: In the diagram below, use similar triangles to relate $x$, the distance from the pole, to $y$, the position of the tip of the shadow. Then relate $\frac{dy}{dt}$ to $\frac{dx}{dt}$.)

(6) Solve for $x$: $8^x = 4$.

## Sections 4.2-4.3, due Monday Oct 30

(1) Compute the first and second derivatives of $y = xe^x$. Tell the intervals on which the curve is (a) increasing, (b) decreasing, (c) concave upward, and (d) concave downward.

(2) Sketch the graph $y = xe^x$, using the results of problem 1. Use your calculator to tell you the value of $y$ when $x = -10$. That should help you in your graphing. Note also that $e^x$ is always positive. Your graph should show clearly all $x$- and $y$-intercepts.

(3) Compute the derivatives of the following functions.

$$y = e^{\sqrt{x+1}}, \qquad y = e^{e^{2x}}.$$

(4) Compute the derivatives of the following functions and tell when each derivative equals 0.

$$y = \frac{e^x+1}{e^x-1}, \qquad y = \frac{x}{e^x}.$$

(5) Compute the first and second derivatives of $y = (1+x)e^{-2x}$. Tell the intervals on which the curve is (a) increasing, (b) decreasing, (c) concave upward, and (d) concave downward.

(6) Sketch the graph $y = (1+x)e^{-2x}$, using the results of problem 5. Your graph should show clearly all $x$- and $y$-intercepts.

**Sections 4.4-4.6, due Friday Nov 3**

(1) Simplify $e^{-2\ln 5}$ and $\ln(x^2y) - \ln(\frac{x}{y})$.

(2) Solve for $x$: (a) $\frac{1}{2}\ln x = 4$, (b) $e^{3x}e^{\ln 2} = 1$, (c) $\ln(x+2) - \ln(x-1) = 1$.

(3) Find the coordinates of the extreme point of $y = e^{2x} - x$, and use the second derivative test to tell whether it is a relative maximum or minimum.

(4) Differentiate (a) $y = \ln(x^3)$, (b) $y = (\ln x)^3$, (c) $y = x^3\ln(x^3+1)$.

(5) Differentiate $y = \ln((x+1)^2(x^2+1)e^{x^2})$.

(6) Use logarithmic differentiation to find $\frac{dy}{dx}$ if

$$y = \frac{(x^2+1)(x+2)^3\sqrt{x-4}}{\sqrt{3x^2+1}}.$$

**Sections 5.1-5.2, due Monday Nov 13**

(1) A bacteria culture with an initial size of 1000 grows exponentially and quadruples in 3 days. (a) Find a formula for the population $P(t)$ after $t$ days. (b) What differential equation does it satisfy? (c) What was its population after 12 hours?

(2) A population is growing exponentially with growth constant .05. In how many years will the population double?

(3) 100 grams of a radioactive substance has decay constant .03, where time is measured in days. Let $P(t)$ denote the amount remaining after $t$ days. (a) Give the formula for $P(t)$. (b) What is the differential equation satisfied by $P(t)$? (c) How much will remain after 5 days? (d) What is the half-life of this radioactive substance?

(4) The decay constant for carbon-14 is .00012, with time measured in years. A parchment has 40% of the level of carbon-14 found in living wood. How old is the parchment?

(5) \$1000 is placed in an account drawing 4.2% yearly interest compounded continuously. Let $A(t)$ denote the amount after $t$ years. (a) What differential equation is satisfied by $A(t)$? (b) What is the formula for $A(t)$? (c) How much will be in the account after three years? (d) How long will it take the account to double? (e) How fast is the balance growing when it doubles?

(6) A \$2000 investment was worth \$5000 after ten years. What rate of interest compounded continuously did this investment earn?

**Sections 5.4-6.1, due Friday Nov 17**

(1) If $y = 4(1 - e^{-3x})$, compute $y'$, and then find numbers $a$ and $b$ such that $y' = a - by$.

(2) Suppose that the number of people who have heard the news after $t$ hours satisfies equations (3) and (4) on page 280. If 1/3 of the people heard the news during the first hour, how much longer *after that* does it take for the *next third* of the people to hear it?

(3) The number of cases of an epidemic after $t$ weeks is $\frac{4000}{1+Be^{-ct}}$. There were initially 10 cases, then five more during the first week. Find the values of $B$ and $c$, and then tell how long will have elapsed when there were 25 cases.

(4) Evaluate the following antiderivatives:

$$\int x^2\sqrt{x}\,dx, \qquad \int \left(x+\tfrac{1}{x}\right)dx, \qquad \int 2e^{-3x}\,dx$$

(5) Find the function $f(x)$ which satisfies $f'(x) = x + e^{-2x}$ and $f(0) = 2$.

(6) A rock is dropped from the top of a 320-foot cliff. Its velocity at time $t$ seconds is $v(t) = -32t$ feet per second.

a. Find $s(t)$, the height of the rock above the ground at time $t$.

b. How long will it take the rock to reach the ground?

c. What will be its velocity an instant before it hits the ground?

**Sections 6.2-6.3, due Monday Nov 27**

(1) Evaluate (a) $\int_0^1 (2e^{-t} - t)\,dt$; (b) $\int_1^9 \frac{x^2+\sqrt{x}}{x}\,dx$ (simplify).

(2) Given that $f'(t) = 2t - e^{2t}$, compute $f(1) - f(-1)$.

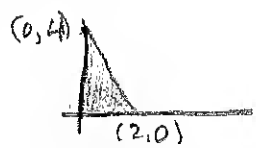
(3) The velocity at time $t$ of an object moving along an axis is $t^2 - 2t$.

a. Compute the net displacement between $t = 0$ and $t = 3$.

b. During what time intervals was it moving forward (resp. backward)?

c. What was its total distance traveled?

(4) Compute the area of the shaded triangle (a) using geometry, and (b) using Theorem 1, integration. You will have to determine the formula for the slanted line.




(5) Use a Riemann sum to approximate the area under $y = x^2$ for $0 \le x \le 2$, using $n = 4$ and midpoints of subintervals. Then compute the actual area, using the Fundamental Theorem of Calculus.

(6) Use a Riemann sum to approximate the area under the graph of $f(x)$ in Figure 14 on page 318 for $1 \le x \le 7$ using $n = 6$ (a) using right endpoints, and (b) using left endpoints.

## Sections 6.4, 6.5, and 8.1, due Friday Dec 1

(1) Find the area of the region bounded by the curve $y = x^2 - 2x - 3$ and the $x$-axis.

(2) Sketch the curves $y = x^2$ and $y = x^3$, and find the area of the region between them.

(3) If an object moves along an axis with velocity $v(t) = t^2 - 4t + 3$, find (a) the displacement between $t = 0$ and $t = 2$, and (b) the total distance traveled between $t = 0$ and $t = 2$.

(4) Find the volume of the solid of revolution obtained by revolving the curve $y = e^x - 1$ about the $x$-axis between $x = 0$ and $x = 1$.

(5) What is the radian measure of the indicated angle?

(6) Construct angles (i.e., pictures such as the one in the previous problem) with radian measure $-\pi/3$ and $7\pi/4$.

## Sections 8.2-8.4, due Friday Dec 8

(1) (a) Draw a set of axes and a line from the origin to the point $(-3, 3)$. (b) Let $t$ denote the radian measure of a counterclockwise angle going directly from the positive $x$-axis to your line. What is the radian measure of $t$? What are $\sin(t)$ and $\cos(t)$? Write these as statements involving the numerical value of $t$. (c) Now do the same thing for the clockwise angle going directly from the positive $x$-axis to your line.

(2) Differentiate $y = \sin^2(2t)$ and $y = \frac{x}{\cos(x^2)}$.

(3) Evaluate $\int \sin(3x)\,dx$ and $\int \cos(\frac{1}{2}(t-1))\,dt$.

(4) Find the area under the curve $y = \cos(2t)$ for $t$ from 0 to $\pi/4$.

(5) Differentiate $y = \tan(2\sqrt{x})$ and $y = \sqrt{\tan(2t)}$.

(6) Find the area under the curve $y = \sec^2(2x)$ for $x$ from 0 to $\pi/6$.